

# On the universality of the spectrum of cosmic rays accelerated at highly relativistic shocks

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## ABSTRACT

I consider analytically particle acceleration at relativistic shocks, in the limit of pitch angle diffusion and large shock Lorentz factors. The derived energy spectral index ( $k = (1 + \sqrt{13})/2 \approx 2.30$ ), and particle pitch angle distribution at the shock are successfully compared with the results of numerical solutions. This totally analytic derivation is completely independent of the detailed dependence of the diffusion coefficient  $D(\mu, p)$  on either parameter, and it is argued on a physical basis that the orientation of the magnetic field is also irrelevant, making  $k$  a universal index.

*Subject headings:* acceleration of particles – shock waves – gamma rays: bursts

## 1. Introduction

There is currently a growing interest in the acceleration of non-thermal particles at highly relativistic shocks. There are three classes of *bona fide* relativistic sources: beyond the well-established extra-Galactic (Blazars) and Galactic (superluminal) sources, both of which exhibit superluminal motions, it is now also well-established that Gamma Ray Bursts (GRBs) display highly relativistic expansions, with Lorentz factors well in excess of 100. Other classes of relativistic sources may include Soft Gamma Ray Repeaters (SGRs), whose recurrent explosions are largely super-Eddington, and special SNe similar to SN 1998bw, which displayed marginally Newtonian expansion ( $\approx 6 \times 10^4 \text{ km s}^{-1}$ ) when optical emission lines became detectable, about a month after the explosion.

With the discovery of GRBs' afterglows, it has now become feasible to derive the energy spectral index  $k$  of electrons accelerated at the forward shock, as a function of the varying (decreasing) shock Lorentz factor  $\gamma$ , provided simultaneous wide-band spectral coverage is available. With the launch of the USA/Italy/UK mission SWIFT, these data will become available for a statistically significant number of bursts, testing directly models for particle acceleration at relativistic shocks. Furthermore, since GRBs must also clearly accelerate protons, the same index  $k$  may determine the spectrum of ultra high energy cosmic rays observed at Earth.

However, until recently, both the lack of astrophysical motivation and the difficulty inherent in treating highly anisotropic distribution functions have stiffened research on this topic. Early

work, both semi-analytic and outright numerical, has concentrated on barely relativistic flows with Lorentz factors of a few, well suited to Blazars and Galactic superluminals, but clearly insufficient for GRBs, the only exception being the numerical simulations of Bednarz and Ostrowski (1998). It is the purpose of this *Letter* to perform an analytic investigation of the large  $\gamma$  limit, to establish which (if any) of the properties of the particles' distribution function depend upon the physical conditions of the fluid.

## 2. The analysis

I deal first with pure pitch angle scattering, and then (Subsection 2.3) I discuss oblique shocks. In the well-known equation for the particles' distribution function, under the assumption of pure pitch angle scattering,

$$\gamma(u + \mu) \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} \left( D(\mu, p)(1 - \mu^2) \frac{\partial f}{\partial \mu} \right), \quad (1)$$

$f$  is computed in the shock frame, in which are also defined the distance from the shock  $z$ , the fluid speed in units of  $c$ ,  $u$ , and fluid Lorentz factor  $\gamma$ . Instead, the scattering coefficient  $D$ , particle momentum  $p$  and particle pitch angle cosine,  $\mu$ , are all defined in the local fluid frame. I make no hypothesis whatsoever about  $D$ , except that it is positive definite and smooth. We place ourselves in the shock frame, and call  $z = 0$  the shock position; the upstream section is for  $z < 0$ , so that the fluid speeds are both  $> 0$ . The above equation admits of an integral: by integrating over  $\mu$  and  $z$  we see that

$$\int_{-1}^1 (u + \mu) f d\mu = \text{const.}, \quad (2)$$

independent of  $z$ . The required boundary condition for  $f$ , *i.e.*, that  $f \rightarrow 0$  as  $z \rightarrow -\infty$ , implies that the constant, upstream, is 0. Downstream, Eq. 2 is also a constant, but, because of Taub's jump conditions, it is not the same constant as upstream. Since the required boundary condition for  $f$  far downstream ( $f_\infty$ ) is that it becomes isotropic and  $f$  is a relativistic invariant, we see that, far downstream,  $\int_{-1}^1 \mu f_\infty d\mu = 0$ . We thus have

$$\int_{-1}^1 (u + \mu) f d\mu = \begin{cases} 0 & z < 0 \\ \int_{-1}^1 u f_\infty d\mu > 0 & z > 0 \end{cases} \quad (3)$$

where the inequality in Eq. 3b (which will become necessary later on) derives from the obvious constraint  $f > 0$ .

## 2.1. Upstream

I begin the analysis by considering Eq. 3a in the limit of very large shock Lorentz factors, in which case, upstream,  $u \rightarrow 1$ . For  $u = 1$ , this reduces to

$$\int_{-1}^1 (1 + \mu) f d\mu = 0 . \quad (4)$$

Since  $1 + \mu > 0$  everywhere in the integration interval except of course at  $\mu = -1$ , and since  $f \geq 0$  we see that, for  $u = 1$  we must have  $f \propto \delta(\mu + 1)$  where  $\delta(x)$  is Dirac's delta. Thus, in this limit, the angular dependence factors out. For reasons to be explained in the next Subsection, we shall also need  $f$  for  $1 - u \ll 1$ , but still  $\neq 0$ . To search for such a solution, we let ourselves be guided by the solution at  $u = 1$ : thus, we let the angular dependence factor out, and use the *Ansatz*  $f = g(z)w((\mu + 1)/h(u))$ . Here  $h(u)$  is an as yet undetermined function of the pre-shock fluid speed such that  $h(u) \rightarrow 0$  as  $u \rightarrow 1$ . In this way, as the speed grows, the angular dependence becomes more and more concentrated toward  $\mu = -1$ , as required by the previously found solution for  $u = 1$ . Introducing our *Ansatz* into Eq. 1 I find

$$\frac{\gamma}{g} \frac{dg}{dz} = \frac{1}{(1 + \mu)w} \frac{d}{d\mu} \left( D(\mu, p)(1 - \mu^2) \frac{dw}{d\mu} \right) = \frac{2D_{-1}\lambda^2}{h^2(u)} , \quad (5)$$

where I defined  $D_{-1} \equiv D(\mu = -1, p)$ , and I factored the eigenvalue for future convenience. Concentrating on the angular part, and defining  $(\mu + 1)/h(u) \equiv y$ , and  $\dot{w} \equiv dw/dy$ ,  $\ddot{w} \equiv d^2w/dy^2$ , I find

$$\frac{D(\mu, p)(1 - \mu^2)}{h^2(u)} \ddot{w} + \frac{\dot{w}}{h(u)} \frac{d}{d\mu} (D(\mu, p)(1 - \mu^2)) - \frac{2\lambda^2 D_{-1}(1 + \mu)w}{h^2(u)} = 0 . \quad (6)$$

We are interested in a solution of the above equation only in the limit  $h(u) \rightarrow 0$ , the only one in which our factored *Ansatz* is a good approximation to the true  $f$ . In this case, the term  $\propto \dot{w}$  is clearly subdominant, and can be neglected in a first approximation (this technique is called dominant balance, Bender and Orszag 1978). Furthermore, for  $h(u) \rightarrow 0$ , we expect  $w(\mu)$  to be more and more concentrated around  $\mu = -1$ , so that in this range we can approximate the term  $D(\mu, p)(1 - \mu^2) \approx 2D_{-1}(1 + \mu)$ , and I obtain  $\ddot{w} \approx \lambda^2 w$  with obvious solution  $w \approx w_0 \exp(-\lambda(\mu + 1)/h(u))$ . The factor  $\lambda/h(u)$  can be determined by inserting this approximate expression for  $w$  into Eq. 3a. A trivial computation yields  $\lambda/h(u) = 1/(1 - u)$ .

Now, going back to the equation for  $g$ , the spatial part of  $f$ , we find, also using the above,

$$\frac{1}{g} \frac{dg}{dz} \approx \frac{2D_{-1}}{\gamma(1 - u)^2} \approx 8\gamma^3 D_{-1} \quad (7)$$

from which, in the end, I find an approximate solution for the distribution function in the limit  $u \rightarrow 1$ :

$$f \approx A \exp(8\gamma^3 D_{-1} z) \exp(-(\mu + 1)/(1 - u)) . \quad (8)$$

It is thus seen that the detailed shape of the pitch angle scattering function  $D(\mu, p)$  is irrelevant, and that what is left of it (its value  $D_{-1}$  at  $\mu = -1$ ) only enters the spatial part of the distribution function  $f$ , not the angular one.

## 2.2. Downstream

We make here the usual assumption, that the distribution function depends upon the particle momentum  $p$  as  $f \propto p^{-s}$  in either frame (but see the Discussion for further comments). From the condition of continuity of the distribution function at the shock, denoting as  $p_a$  and  $\mu_a$  the particle's momentum and cosine of the pitch angle in the downstream frame, we have

$$\frac{1}{p_a^s} w_a(\mu_a) \propto \frac{1}{p^s} \exp(-(\mu + 1)/(1 - u)) \quad (9)$$

where the irrelevant constant of proportionality does not depend on  $p, p_a, \mu, \mu_a$ . Using the Lorentz transformations to relate  $p, p_a, \mu, \mu_a$  ( $\mu = (\mu_a - u_r)/(1 - u_r \mu_a)$ ,  $p = p_a \gamma_r (1 - u_r \mu_a)$ , with  $u_r$  and  $\gamma_r$  the relative speed and corresponding Lorentz factor between the upstream and downstream fluids), I find

$$w_a(\mu_a) = \frac{1}{(1 - u_r \mu_a)^s} \exp\left(-\frac{(\mu_a + 1)(1 - u_r)}{(1 - u)(1 - u_r \mu_a)}\right). \quad (10)$$

For  $u \rightarrow 1$ , it is easy to derive from Taub's conditions (Landau and Lifshitz 1987) that  $u_r \rightarrow 1$ , and that  $(1 - u_r)/(1 - u) \approx \gamma^2/\gamma_r^2 \rightarrow 2$ . This result does use a post-shock equation of state  $p = \rho/3$ , which is surely correct in the limit  $u \rightarrow 1$ . In the end, I obtain

$$w_a(\mu_a) = \frac{1}{(1 - \mu_a)^s} \exp\left(-2\frac{\mu_a + 1}{1 - \mu_a}\right). \quad (11)$$

This equation shows why we needed to determine the pitch angle distribution, in the upstream frame, even for  $1 - u \neq 0$ : in fact, even though the angular distribution in the upstream frame (Eq. 8) tends to a singularity, the downstream distribution does not (because the factor  $(1 - u)/(1 - u_r)$  has a finite, non-zero limit), and the concrete form to which it tends depends upon the departures of the upstream distribution from a Dirac's delta.

From now on I will drop the subscript  $a$  in  $\mu_a$ , since all quantities refer to downstream. In order to determine  $s$ , we now appeal to a necessary regularity condition which must be obeyed by the initial (*i.e.*, for  $z = 0$ ) pitch angle distribution, Eq. 11. Looking at Eq. 1 specialized to the downstream case, where  $u = 1/3$  for very fast shocks, we see that this equation has a singularity at  $\mu = -1/3$ . Passing through this singularity will fix the index  $s$ . It is not convenient to use  $f$  directly; rather, I use its Laplace transform

$$\hat{f}(r, \mu) \equiv \int_0^{+\infty} f(z, \mu) e^{-rz} dz. \quad (12)$$

Taking Laplace transforms of both sides of Eq. 1 I obtain

$$-\frac{\gamma(1/3 + \mu)w_a(\mu)}{r} + \gamma(1/3 + \mu)\hat{f} = \frac{1}{r} \frac{\partial}{\partial \mu} \left( D(\mu, p)(1 - \mu^2) \frac{\partial \hat{f}}{\partial \mu} \right). \quad (13)$$

I am interested in the limit  $r \rightarrow +\infty$ . In fact, here I can use two results. First, in this limit, it is well-known (Watson's Lemma, Bender and Orszag 1978) that Eq. 12 reduces to

$$\hat{f}(r, \mu) \rightarrow \frac{f(z = 0, \mu)}{r} = \frac{w_a(\mu)}{r}. \quad (14)$$

Despite this wonderful result in all its generality, I will actually use it only in the neighborhood of  $\mu = -1/3$ ; here, Eq. 13 takes on a simple form: defining  $t \equiv \mu + 1/3$ ,

$$\frac{bt}{r} + at\hat{f} = \frac{1}{r} \frac{\partial^2 \hat{f}}{\partial t^2} + \frac{c}{r} \frac{\partial \hat{f}}{\partial t} \quad (15)$$

where I defined  $b \equiv (\gamma w_a(\mu)D(\mu, p)(1 - \mu^2))|_{\mu=-1/3}$ ,  $a \equiv (\gamma D(\mu, p)(1 - \mu^2))|_{\mu=-1/3}$ , and  $c \equiv (\partial/\partial\mu D(\mu, p)(1 - \mu^2))/D(\mu, p)(1 - \mu^2)|_{\mu=-1/3}$ . Now I make the *Ansatz* (to be checked *a posteriori*) that the term  $\propto \partial\hat{f}/\partial t$  is negligible compared to the second order derivative in the limit  $r \rightarrow +\infty$ . I am interested only in the most significant term in  $r$ , since Eq. 14 was only obtained to this order. Then, the equation 15 becomes

$$at\hat{f} = \frac{1}{r} \frac{\partial^2 \hat{f}}{\partial t^2} . \quad (16)$$

The above equation is the prototype of the one–turning point problem. Its solution, strictly in the neighborhood of the point  $t = \mu + 1/3 = 0$ , is (Bender and Orszag 1978, Sect. 10.4, Eq. 10.4.13b):

$$\hat{f}(t) \approx r^{1/12} C Ai(r^{1/3} at) \quad (17)$$

where  $C$  is an arbitrary constant, and  $Ai(x)$  is one of Airy’s functions. From this it can easily be checked that our *Ansatz* was justified.

Clearly, close to the point  $t = \mu + 1/3 = 0$ , Eq. 14 and Eq. 17 must give the same results. Thus I find that, close to  $\mu = -1/3$ ,

$$w_a(\mu) \propto Ai(r^{1/3} a(\mu + 1/3)) \quad (18)$$

which solves our problem: from this in fact we see that, since  $d^2 Ai(x)/dx^2 = x Ai(x)$  by definition, and thus  $d^2 Ai(x)/dx^2 = 0$  in  $x = 0$ , then we must have

$$\frac{\partial^2 w_a}{\partial \mu^2} \Big|_{\mu=-1/3} = 0 . \quad (19)$$

This is our sought for extra condition for  $s$ ; we have seen that it comes directly from demanding that the boundary condition of the problem, Eq. 11, manages to pass through the singular point of Eq. 1, which I showed to be a conventional one–turning point problem familiar from elementary quantum mechanics.

By substituting into Eq. 11 I find

$$\left( \frac{\partial^2 w_a}{\partial \mu^2} \right)_{\mu=-1/3} = 2^{-2(2+s)} 3^{2+s} e^{-1} (s^2 - 5s + 3) = 0 \quad (20)$$

from which we obtain  $s = (5 \pm \sqrt{13})/2$ . The solution with the minus sign is unacceptable: in fact, if we plug Eq. 11 into the conservation equation 2, we see (Fig. 1) that for  $s \leq 3$  the integral is  $\leq 0$ . We remarked after Eq. 3b, however, that this integral had necessarily to be  $> 0$ , so that we may conclude that 3 is an absolute lower limit to  $s$ . Thus we discard the solution with the minus sign, and are left with the unique solution

$$s = \frac{5 + \sqrt{13}}{2} \approx 4.30 . \quad (21)$$

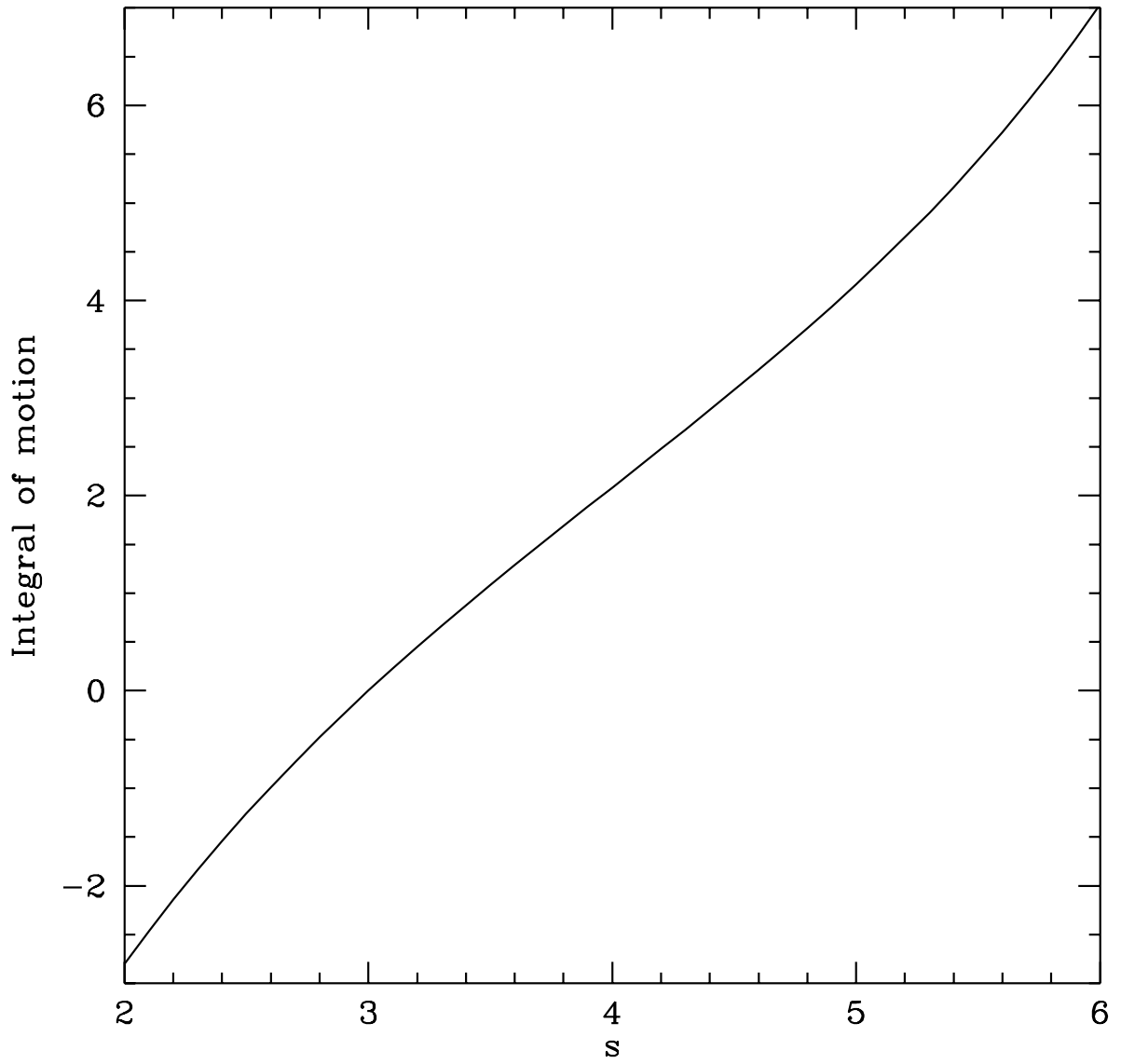


Fig. 1.— The integral of Eq. 3b with  $u = 1/3$ , for the angular distribution of Eq. 11, as a function of the parameter  $s$ , with arbitrary vertical scale.

### 2.3. Oblique shocks

Let us call  $\phi$  the angle that the magnetic field makes with the shock normal, in the upstream fluid. Then shocks can be classified as either subluminal or superluminal, depending upon whether, upstream,  $u/\cos\phi < 1$  or  $u/\cos\phi > 1$ , respectively (de Hoffmann and Teller 1950). We are interested in the limit  $u \rightarrow 1$ , so that most shocks will be of the superluminal type. In this case, we could (but we won't) move to a frame where the magnetic field is parallel to the shock surface, both upstream and downstream. However, downstream this extremely orderly field configuration appears more idealized than warranted by physical reality and observations. In fact, behind a relativistic shock, a large number of processes (compression, shearing, turbulent dynamo, Parker instability, two-stream instability) can generate magnetic fields; furthermore, there is no obvious reason why these fields should have large coherence lengths. In GRBs, a large number of observations of different afterglows supports this picture, the most detailed of all being those of GRB 970508, extending from a few hours to 400  $d$  after the burst (Waxman, Frail and Kulkarni 1998; Frail, Waxman and Kulkarni 2000, and references therein). Accurate and successful modeling fixes the post-shock ratio of magnetic to non-magnetic energy densities to  $\epsilon_B \approx 0.1$ . Notice that here the protons' rest mass is not even the largest contribution to the non-magnetic energy density! Polarization measurements also support, albeit less cogently, the idea of a small coherence length: of the four bursts observed so far, only one has a detected polarization, at the 1.7% level (GRB 990510, Covino *et al.*, 1999).

Thus the most plausible physical model downstream, is that particles move in a locally generated turbulent, dynamically negligible magnetic field; if then we call  $l$  the average post-shock field coherence length, and restrict our attention to particles with sufficiently large energies (*i.e.*, with gyroradii  $r_g > l$ ), we see there can be no reflection as particles approach the shock from downstream. It follows that we expect the situation downstream to be identical to that of pure pitch angle scattering. Upstream, the parallel magnetic field is also irrelevant. In fact, backward deflection of a particle occurs on a length-scale  $r_g$ , but backward diffusion of the particle by magnetic irregularities only requires the sideways deflection by an angle  $\approx \gamma^{-2}$  ( $\gamma$  being the shock Lorentz factor), for the shock to overrun the particle. This typically occurs on a length  $\eta r_g/\gamma^2$ , with  $\eta \approx$  a few. So, as  $\gamma \rightarrow \infty$ , the length-scale for scattering upstream by the magnetic field increases, while that by magnetic irregularities decreases: the field is irrelevant. In the end, the same analysis as for pure pitch angle scattering applies, and the same index  $s$  and pitch angle distributions at the shock follow.

In the case of subluminal shocks, a similar comment applies. Downstream, we expect on a physical basis the same situation as for superluminal shocks. Upstream, Eq. 1 is replaced by (Kirk and Heavens 1989)

$$\gamma \cos\phi(u + \mu) \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} \left( D(\mu, p)(1 - \mu^2) \frac{\partial f}{\partial \mu} \right). \quad (22)$$

to which the same analysis as in Subsection 2.1 can be applied. Thus we find the same  $s$  and pitch angle distributions at the shock as above.

As a corollary, it may be noticed that the above argument also implies that the results above are independent of the ratio  $\kappa_{\perp}/\kappa_{\parallel}$ , the cross-field and parallel diffusion coefficients.

### 3. Discussion

For ultrarelativistic particles the energy spectral index  $k$  is related to  $s$  by

$$k = s - 2 = \frac{1 + \sqrt{13}}{2} \approx 2.30, \quad (23)$$

which is our final result. Also, now that we know  $s$ , the final pitch angle distribution at the shock, but downstream, can be determined (Eq. 11), and is plotted in Fig. 2. None of these results depends upon the specific form of  $D(\mu, p)$ , so that widely differing assumptions should yield precisely the same results.

How does this compare with numerical work? The near-constancy of the index  $s$  (or  $k$ ) explains moderately well the results of previous authors: Kirk and Schneider (1987) find  $s = 4.3$  for their computation with the highest speed, which is however a modest Lorentz factor of  $\gamma = 5$ , and a single functional dependence of  $D$  on  $\mu$ . Heavens and Drury (1988) find again a result of  $s = 4.2$  for equally modest Lorentz factors, but for two different recipes for  $D$ . Extensive numerical computations using a MonteCarlo technique (*i.e.*, totally independent of the validity of Eq. 1) were performed by Bednarz and Ostrowski (1998), for a wide variety of assumptions about the scattering properties of the fluids. They remarked quite explicitly that the energy spectral index  $k$  seemed to converge to a constant value, independent of shock Lorentz factor (provided  $\gamma \gg 1$ ), magnetic field orientation angle  $\phi$ , and diffusion coefficient ratio,  $\kappa_{\perp}/\kappa_{\parallel}$ . They found  $k \approx 2.2$ . The present work confirms (for all untried forms for  $D(\mu, p)$ ) and extends their simulations (by yielding the exact value, and explicit forms for the particle angular distributions). The upstream angular distributions also agree well: Fig.3a of Bednarz and Ostrowski clearly shows that this is (for the highest displayed value of  $\gamma = 27$ ) a Dirac's delta, in agreement with the large- $\gamma$  limit in Section 2.1. However, the downstream pitch angle distributions (in their Fig. 3b) agree well with mine, but not perfectly. Gallant, Achterberg and Kirk (1998) have claimed that there is a small error in Bednarz and Ostrowski's distributions. As a matter of fact, my distribution (Fig. 2) agrees much better with Gallant *et al.*'s and Kirk and Schneider's (1987) than Bednarz and Ostrowski's, despite the very small shock Lorentz factors of these two papers ( $\gamma = 2.3$  and  $\gamma = 5$ , respectively). Possibly, the small error in question may even explain the (small!) discrepancy between the two values of  $k$ .

A limitation applies to the claim of universality of Eqs. 8, 11 and 23: I neglected any process altering the particles' energy during the scattering. Clearly, the results of this paper only apply in the limit  $\delta p/p \lesssim 1$ , where  $\delta p$  is the typical momentum transfer in each scattering event. In the large momentum limit considered here, it seems unlikely that this constraint may be violated.

Lastly, a comment on the assumed dependence  $\propto p^{-s}$  of the distribution function upon particle



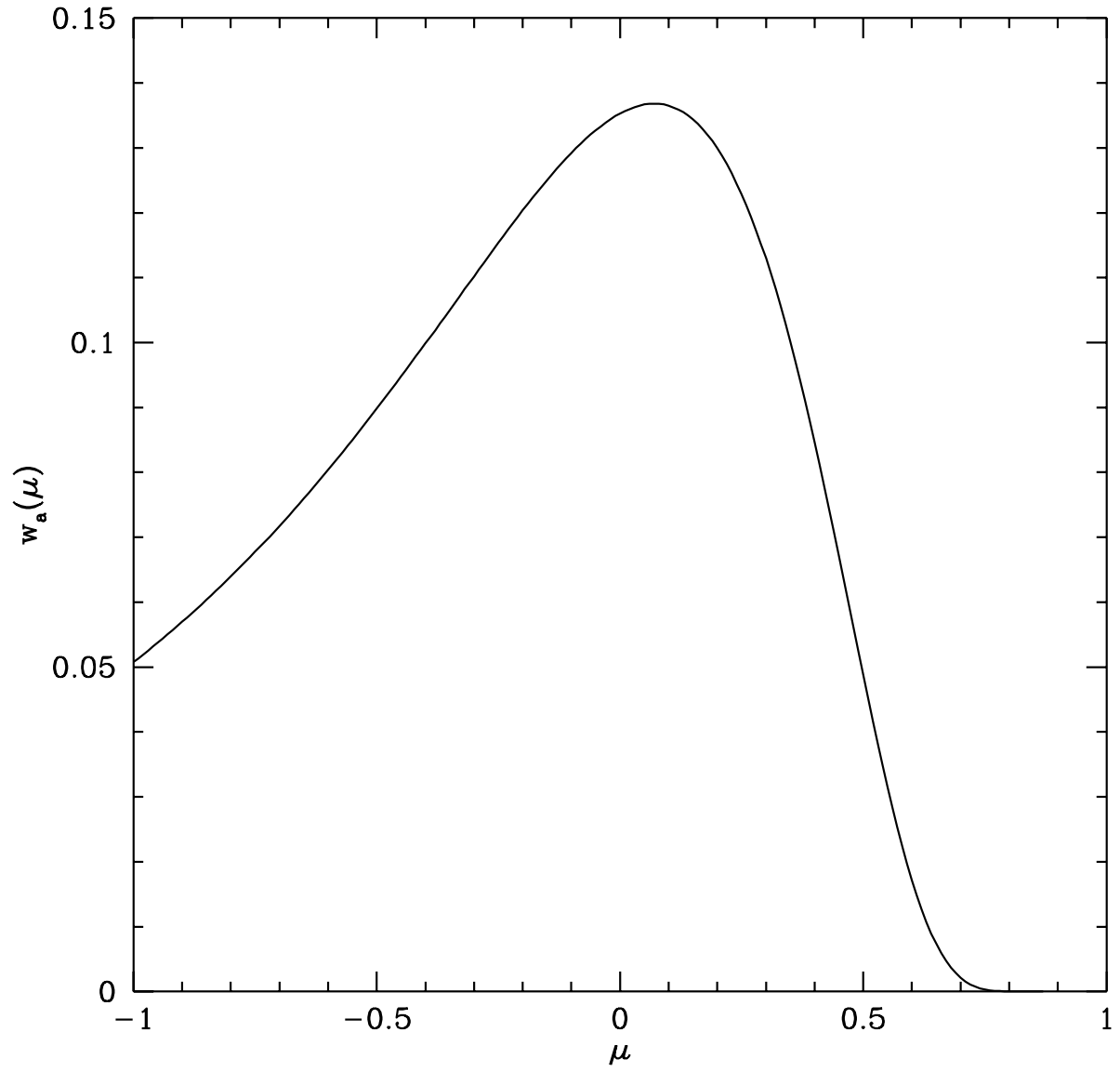


Fig. 2.— Pitch angle distribution at the shock in the downstream frame, Eq. 11 with  $s$  from Eq. 21, with arbitrary vertical scale.

momenta is in order. It can be seen from Eq. 1 that such a dependence is *not* required by this equation. To see this, let us make the usual assumption that  $D$  is homogeneous of degree  $-r$  in  $p$ , *i.e.*,  $D(\mu, p) = q(\mu)p^{-r}$ . Then by defining a new variable  $\hat{z} \equiv z/p^r$ , we see that the form assumed by Eq. 1 after this change of variable is identical to the original one, except that now  $p$  has altogether disappeared. At large  $z$  (*i.e.*, far downstream),  $f \rightarrow f_\infty = \text{constant}$ , and there is no  $p$ -dependence. This paradox is solved by noticing that the real problem to be solved involves both scattering (= Fermi acceleration) and injection. In this case, a typical injection momentum  $p_0$  arises naturally, and the dimensional problem discussed above is naturally solved: we must have  $f = f(\dots, p/p_0, \dots)$  where the dots indicate all other parameters. In the limit of  $p_0 \rightarrow 0$ ,  $f$  *does not* tend to a constant independent of  $p_0$  as is always assumed, but tends instead to zero as  $f \rightarrow (p_0/p)^s$ . Problems of this sort, though rare in astrophysics, are common in hydrodynamics, where they are called self-similar problems of the second kind (Zel'dovich 1956). They range from the deceptively simple laminar flow of an ideal fluid past an infinite wedge (Landau and Lifshitz 1987) to the illuminating case of the filtration in an elasto-plastic porous medium (Barenblatt 1996). It is remarkable that, in the problem at hand, no such complication is necessary to fix the all-important index  $s$ , yet the powerful methods of intermediate asymptotics (Barenblatt 1996) and the renormalization group (Goldenfeld 1992) can be brought to bear on the intermediate  $\gamma$  cases, where no easy limiting solution can be found.

In short, what I have done in this paper is to show that the spectrum of non-thermal particles accelerated at relativistic shocks is universal, in the sense that the energy spectral index  $k$ , and the angular distributions in both the upstream and downstream frames (Eqs. 8, 11, 23, and Fig. 2) do not depend upon the scattering function  $D(\mu, p)$ , the shock Lorentz factor (provided of course  $\gamma \gg 1$ ), the magnetic field geometry, and the ratio of cross-field to parallel diffusion coefficients. Thus we have the result that the cosmic rays' spectra are independent of flow details in both the Newtonian (Bell 1978) and the relativistic regimes.

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